AMS 250: An Introduction to High Performance Computing

Parallel Performance Theory

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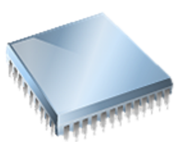
Outline

- Performance and Scalability
- Analytical performance measures
- Amdahl’s law and Gustafson’s law

What is Performance?

- In computing, performance is defined by 2 factors
  - Computational requirements
  - Computing resources
- Computational problems translate to requirements
- Computing resources:



Hardware

Time

Energy

… and ultimately

Money

$$Performance \sim \frac{1}{Resources\ for\ solution}$$

Why do we care about Performance?

- Performance itself is a measure of how well the computational requirements can be satisfied
- We evaluate performance to understand the relationships between requirements and resources
  - Decide how to change "solutions" to target objectives
- Performance measures reflect decisions about how and how well "solutions" are able to satisfy the computational requirements

> *"The most constant difficulty in contriving the engine has arisen from the desire to reduce the time in which the calculations were executed to the shortest which is possible."*
>
> *Charles Babbage, 1791 – 1871*

What is Parallel Performance?

- Here we are concerned with performance issues when using a parallel computing environment
  - Performance with respect to parallel computation
- Performance is the *raison d'être* for parallelism
  - Parallel performance versus sequential performance
  - If the "performance" is not better, parallelism is not necessary
- *Parallel processing* includes techniques and technologies necessary to compute in parallel
  - Hardware, networks, operating systems, parallel libraries, languages, compilers, algorithms, tools, …
- Parallelism must deliver performance
  - How? How well?

Performance Expectation (Loss)

- If each processor is rated at *k* MFLOPS and there are *p* processors, should we see *k*p* MFLOPS performance?
- If it takes 100 seconds on 1 processor, shouldn't it take 10 seconds on 10 processors?
- Several causes affect performance
    - Each must be understood separately
    - But they interact with each other in complex ways
        - Solution to one problem may create another
        - One problem may mask another
- Scaling (system & problem size) can change conditions

Embarrassingly Parallel Computations

- An embarrassingly parallel computation is one that can be obviously divided into completely independent parts that can be executed simultaneously
  - In a truly embarrassingly parallel computation there is no interaction between separate processes
  - In a nearly embarrassingly parallel computation results must be distributed and collected/combined in some way
- Embarrassingly parallel computations have potential to achieve maximal speedup on parallel platforms
  - If it takes *T* time sequentially, there is the potential to achieve *T/P* time running in parallel with *P* processors
  - What would cause this not to be the case always?

Performance and Scalability

- Evaluation
  - *Sequential* runtime (T_{seq}) is a function of
    - problem size and architecture
  - *Parallel* runtime (T_{par}) is a function of
    - problem size and parallel architecture
    - # of processors used in the execution
  - Parallel performance affected by
    - algorithm + architecture
- Scalability
  - Ability of parallel algorithm to achieve performance gains proportional to the number of processors and the size of the problem

Performance Metrics and Formulas

- T_1 is the execution time on a single processor
- T_p is the execution time on a p processor system
- S_p is the *speedup*

$$S_p = \frac{T_1}{T_p}$$

- E_p is the *efficiency*

$$E_p = \frac{S_p}{p}$$

- C_p is the *cost*

$$C_p = p \times T_p$$

- Parallel algorithm is *cost-optimal,* if *parallel time = sequential time*

  $C_p = T_1$ & $E_p = 100\%$

Amdahl's Law

- Fixed problem size
- Let f be the fraction of a program that is sequential, then $1\text{-}f$ is the fraction that can be parallelized

$$T_1 = f\,T_1 + (1\text{-}f)\,T_1$$

$$T_p = f\,T_1 + (1\text{-}f)\,T_1/p$$

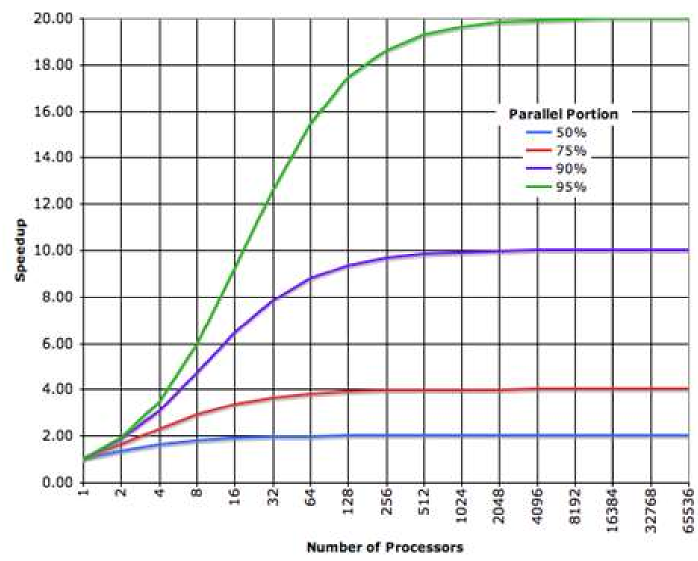
$$S_p = T_1 \;/\; T_p$$

$$= T_1 \;/\; (fT_1 + (1\text{-}f)T_1\,/p))$$

$$= 1 \;/\; (f \;+ (1\text{-}f)/p))$$

- As $p \rightarrow \infty$

$$S_p = 1 \;/\; f$$



Amdahl's Law and Scalability

- Scalability
  - Ability of parallel algorithm to achieve performance gains proportional to the number of processors and the size of the problem
- When does Amdahl's Law apply?
  - When the problem size is fixed
  - *Strong scaling*
  - Speedup bound is determined by the degree of sequential execution time in the computation, not # of processors!

    $$p \to \infty,\ S_p = S_\infty \to 1/f$$
  - Uhh, this is not good … Why?
  - Perfect efficiency is hard to achieve

http://www-inst.eecs.berkeley.edu/~n252/paper/Amdahl.pdf

Gustafson-Barsis's Law

- We often increase the size of problems to fully exploit the available computing power
- Let f be the fraction of a program that is sequential, then $f_{par}=1-f$ is the fraction that can be parallelized. The workload on one processor is:

  $$W_1 = f\,W_1 + (1-f)\,W_1$$

- Assuming parallel time is kept constant, the total workload on p processors is:

  $$W_p = f\,W_1 + (1-f)\,W_1 * p$$

- Scaled speedup:

  $$\begin{aligned} S_p &= W_p \;/\; W_1 \\ &= f + (1-f) * p \\ &= 1-f_{par} + f_{par} * p = 1 + (p-1)\,f_{par} \end{aligned}$$

- Efficiency: $E_p = S_p \;/\; p = (1-f_{par})/p + f_{par}$

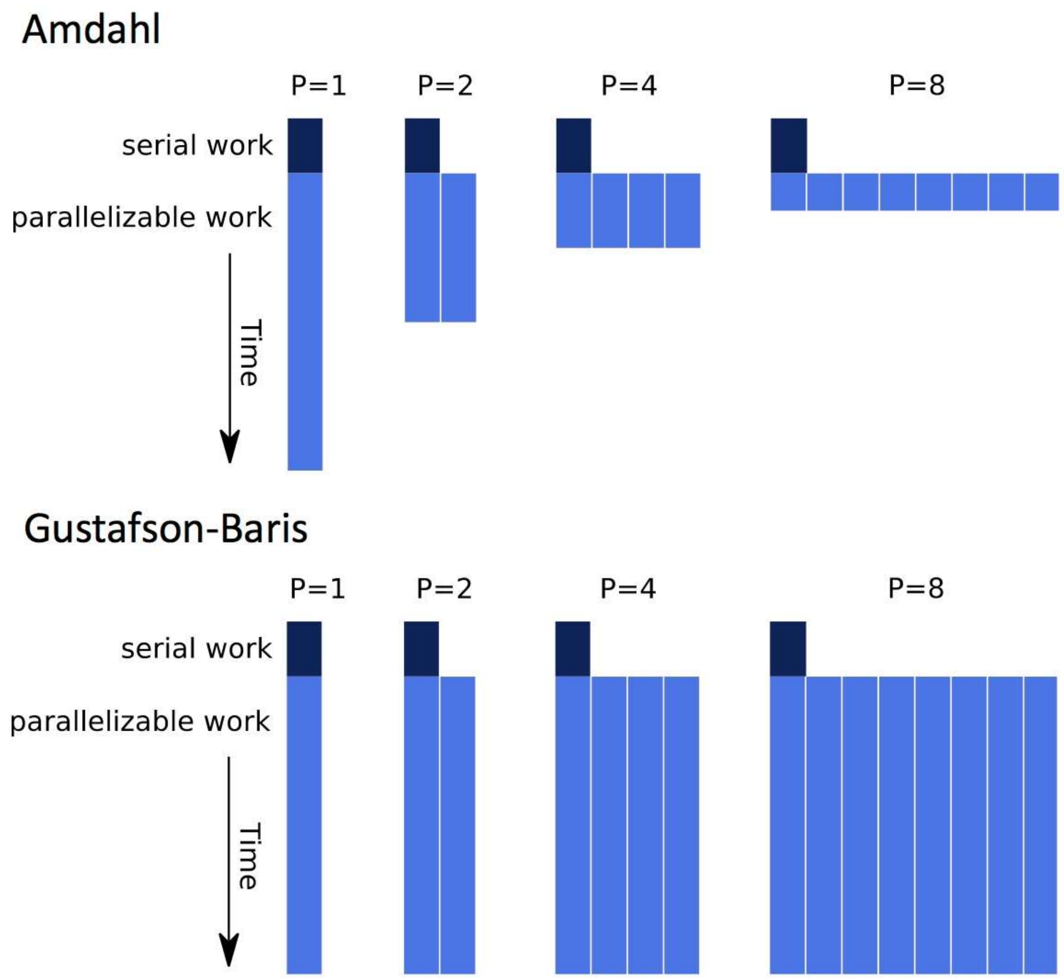
Gustafson-Barsis's Law and Scalability

- Scalability
  - Ability of parallel algorithm to achieve performance gains proportional to the number of processors and the size of the problem
- When does Gustafson-Barsis's Law apply?
  - When the problem size can increase as the number of processors increases
  - *Weak scaling*
  - Speedup function includes the number of processors!

    $S_p = 1 + (p-1)f_{par}$
  - Can maintain or increase parallel efficiency as the problem scales

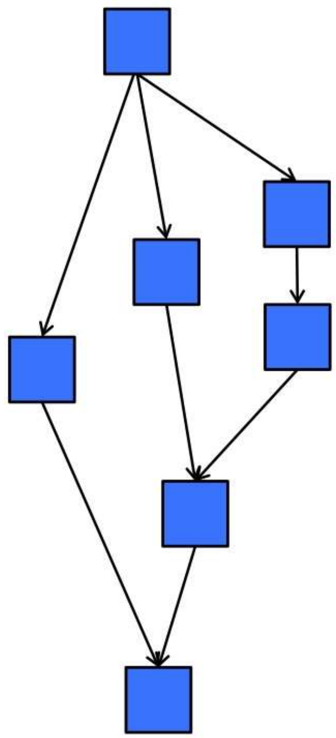
http://www.johngustafson.net/pubs/pub13/amdahl.htm

Amdahl versus Gustafson-Barsis



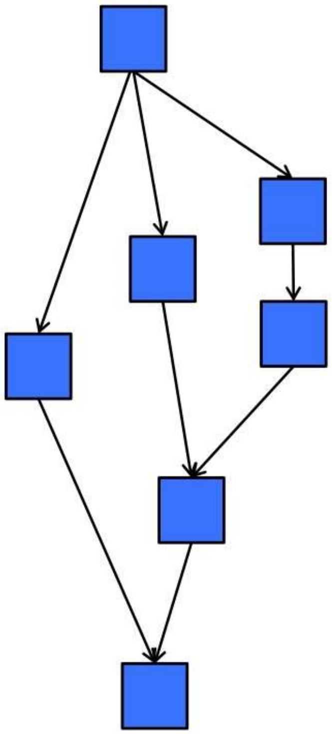
DAG Model of Computation

- Think of a program as a directed acyclic graph (DAG) of tasks
  - A task can not execute until all the inputs to the tasks are available
  - These come from outputs of earlier executing tasks
  - DAG shows explicitly the task dependencies
- Think of the hardware as consisting of workers (processors)
- Consider a *greedy* scheduler of the DAG tasks to workers
  - No worker is idle while there are tasks still to execute



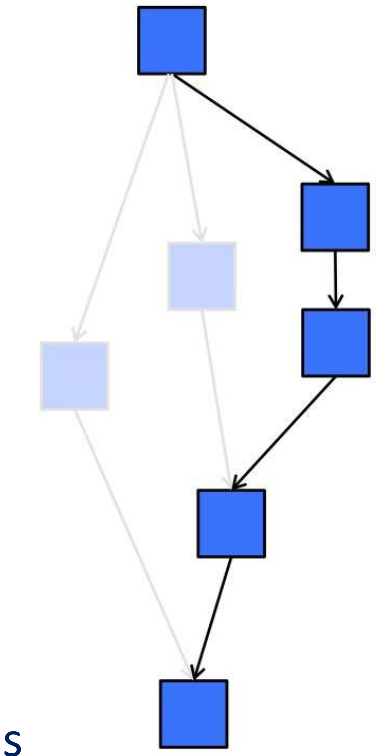
Work-Span Model

- T_P = time to run with P workers
- T_1 = *work*
  - Time for serial execution
    - execution of all tasks by 1 worker
  - Sum of all work
- T_∞ = *span*
  - Time along the *critical path*
- Critical path
  - Sequence of task execution (path) through DAG that takes the longest time to execute
  - Assumes an infinite # workers available



Work-Span Example

- Let each task take 1 unit of time
- DAG at the right has 7 tasks
- $T_1 = 7$
  - All tasks have to be executed
  - Tasks are executed in a serial order
  - Can they execute in any order?
- $T_\infty = 5$
  - Time along the *critical path*
  - In this case, it is the longest path length of any task order that maintains necessary dependencies



Lower/Upper Bound on Greedy Scheduling

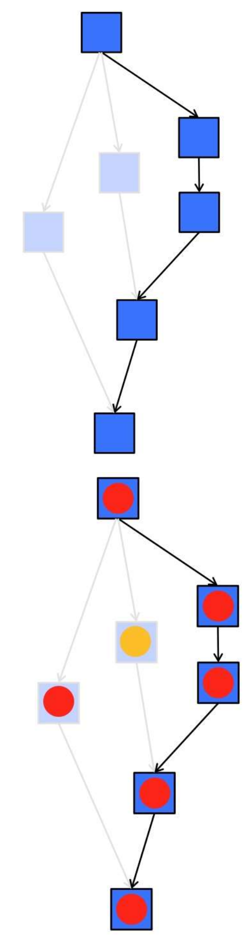
- Suppose we only have P workers
- We can write a work-span formula to derive a lower bound on T_p

$$Max(T_1 / p , T_\infty) \leq T_P$$

- T_∞ is the best possible execution time
- **Brent's Lemma** derives an upper bound
  - Capture the additional cost executing the other tasks not on the critical path
  - Assume we can do so without overhead

$$T_p \leq (T_1 - T_\infty) / p + T_\infty$$

$$\boxed{\boldsymbol{Max(T_1 / p , T_\infty) \leq T_p \leq (T_1 - T_\infty) / p + T_\infty}}$$



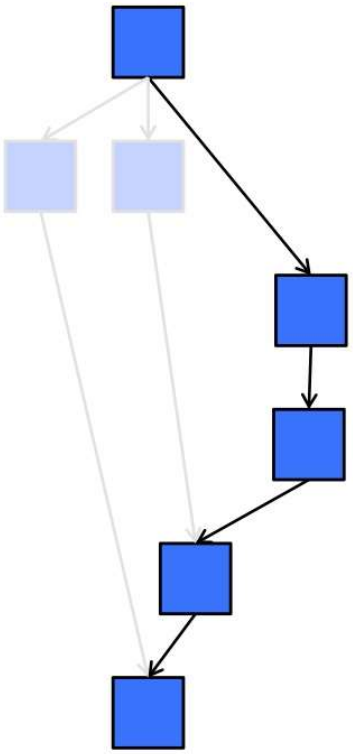
https://maths-people.anu.edu.au/~brent/pub/pub022.html

Consider Brent's Lemma for 2 Processors

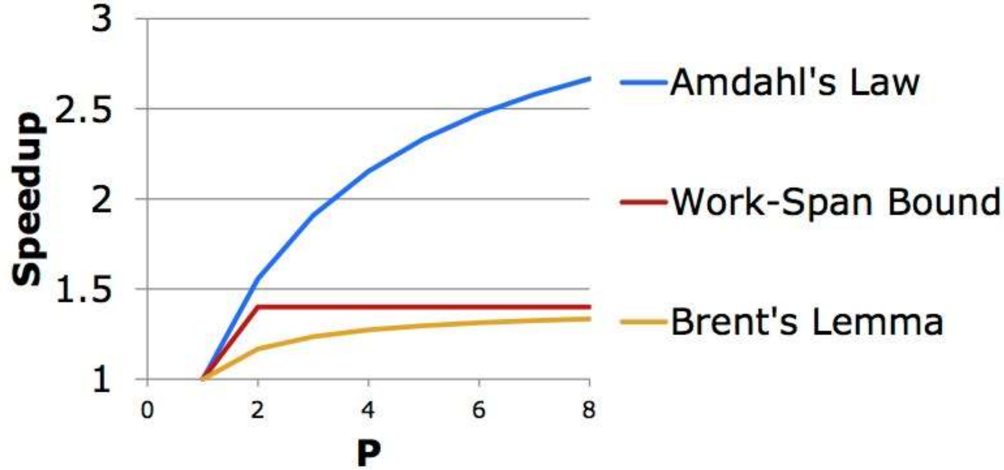
- $T_1 = 7$
- $T_\infty = 5$
- $T_2 \quad \leq (T_1 - T_\infty) / \mathrm{P} + T_\infty$

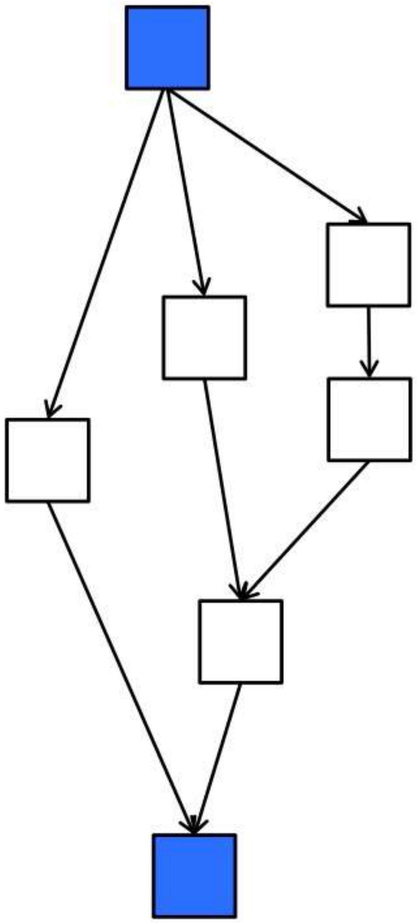
  $\leq (7 - 5) / 2 + 5$

  ≤ 6



Amdahl was an optimist!





f = 2/7, $T_1 = 7$, $T_\infty = 5$

Amdahl's Law: $S_p = 1 / (f + (1-f)/p)) = 7p/(5+2p)$

$Max(T_1 / p , T_\infty) \le T_p \le (T_1 - T_\infty) / p + T_\infty$

$\Rightarrow Max(7/p, 5) \le T_p \le 2/p + 5$

$\Rightarrow 7p/(2+5p) \le S_p \le 7/ Max(7/p, 5)$

Estimating Running Time

- Scalability requires that T_∞ be dominated by T_1

$$Max(T_1 / p \,,\, T_\infty) \le T_p \le (T_1 - T_\infty) / p + T_\infty$$
$$\Rightarrow T_p \approx T_1 / p + T_\infty \;\; if \; T_\infty \ll T_1$$

- Increasing work (T_1) hurts parallel execution proportionately
- The span (T_∞) impacts scalability, even for finite p

Parallel Slack

- Sufficient parallelism implies linear speedup

$$\underbrace{T_P \approx T_1/P}_{\text{Linear speedup}} \quad \text{if} \quad \underbrace{T_1/T_\infty \gg P}_{\textit{Parallel slack}}$$

- Is it possible to have superlinear speedup ($S_p > p$)?