**AMS 206B:** Intermediate Bayesian Inference
**HOMEWORK # 3 (Noninformative Priors and additional problems on Bayesian calculations)**

1. Let $(X_1, X_2, X_3)$ have trinomial distribution with density

$$f(x_1, x_2, x_3|\theta_1, \theta_2) \propto \theta_1^{x_1}\theta_2^{x_2}(1 - \theta_1 - \theta_2)^{x_3}.$$

   Derive Jeffreys prior for $(\theta_1, \theta_2)$.

2. Robert Problem 3.9.

3. Robert Problem 3.10.

4. Robert Problem 3.31.

5. Robert Problem 3.56.

6. Let $y_t = \rho y_{t-1} + \epsilon_t$, $\epsilon_t \sim^{i.i.d.} N(0, \sigma^2)$. This is a popular model in time series analysis known as the autoregressive model of order one or AR(1).

   (a) Write down the conditional likelihood given $y_1$, i.e., $f(y_2, \ldots, y_n|y_1, \rho, \sigma^2)$.

   (b) Assume a prior of the form $\pi(\rho, \sigma^2) \propto 1/\sigma^2$.

      i. Find the joint posterior $p(\rho, \sigma^2|y_1, \ldots, y_n)$ based on the conditional likelihood.

      ii. Find $p(\rho|\sigma^2, y_1, \ldots, y_n)$ and $p(\sigma^2|y_1, \ldots, y_n)$ based on the conditional likelihood.

      iii. Simulate two data sets with $n = 500$ observations each. One with $\rho = 0.95, \sigma^2 = 4$ and another one with $\rho = 0.3, \sigma^2 = 4$. Fit the model above to the two data sets. Summarize your posterior results in both cases.